

Exercise 31

Find an equation of the tangent line to the given curve at the specified point.

$$y = \frac{x^2 - 1}{x^2 + x + 1}, \quad (1, 0)$$

Solution

Start by finding the slope of y at $x = 1$. Evaluate the derivative using the quotient rule.

$$\begin{aligned} y' &= \frac{d}{dx} \left(\frac{x^2 - 1}{x^2 + x + 1} \right) \\ &= \frac{\left[\frac{d}{dx}(x^2 - 1) \right] (x^2 + x + 1) - \left[\frac{d}{dx}(x^2 + x + 1) \right] (x^2 - 1)}{(x^2 + x + 1)^2} \\ &= \frac{(2x)(x^2 + x + 1) - (2x + 1)(x^2 - 1)}{(x^2 + x + 1)^2} \\ &= \frac{x^2 + 4x + 1}{(x^2 + x + 1)^2} \end{aligned}$$

Evaluate it at $x = 1$.

$$y'(1) = \frac{6}{9} = \frac{2}{3}$$

Therefore, the equation of the line with slope $2/3$ that goes through $(1, 0)$ is

$$y - 0 = \frac{2}{3}(x - 1).$$

