Exercise 31

Find an equation of the tangent line to the given curve at the specified point.

$$y = \frac{x^2 - 1}{x^2 + x + 1}, \quad (1,0)$$

Solution

Start by finding the slope of y at x = 1. Evaluate the derivative using the quotient rule.

$$y' = \frac{d}{dx} \left(\frac{x^2 - 1}{x^2 + x + 1} \right)$$

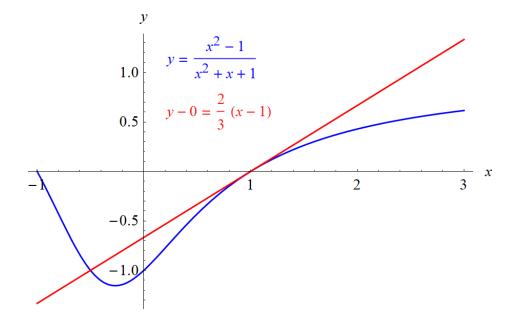
= $\frac{\left[\frac{d}{dx} (x^2 - 1) \right] (x^2 + x + 1) - \left[\frac{d}{dx} (x^2 + x + 1) \right] (x^2 - 1)}{(x^2 + x + 1)^2}$
= $\frac{(2x)(x^2 + x + 1) - (2x + 1)(x^2 - 1)}{(x^2 + x + 1)^2}$
= $\frac{x^2 + 4x + 1}{(x^2 + x + 1)^2}$

Evaluate it at x = 1.

$$y'(1) = \frac{6}{9} = \frac{2}{3}$$

Therefore, the equation of the line with slope 2/3 that goes through (1,0) is

$$y - 0 = \frac{2}{3}(x - 1).$$



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